

# Nonsingular Universes in Gauss-Bonnet Gravity's Rainbow

Seyed Hossein Hendi<sup>1,2\*</sup>, Mehrab Momennia<sup>1†</sup>, Behzad Eslam Panah<sup>1‡</sup>, Mir Faizal<sup>3§</sup>

<sup>1</sup> *Physics Department and Biruni Observatory, College of Sciences, Shiraz University, Shiraz 71454, Iran*

<sup>2</sup> *Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), Maragha, Iran*

<sup>3</sup> *Department of Physics and Astronomy, University of Lethbridge, Lethbridge, AB T1K 3M4, Canada*

In this paper, we will study the rainbow deformation of the FRW cosmology in both Einstein gravity and Gauss-Bonnet gravity. We will demonstrate that the singularity in the FRW cosmology can be removed because of the rainbow deformation of the FRW metric. We will obtain the general constraints required for the FRW cosmology to be free from singularities. It will be observed that the inclusion of Gauss-Bonnet gravity can significantly change the constraints required to obtain a nonsingular universes. We will use a rainbow functions motivated from the hard spectra of gamma-ray bursts to deform the FRW cosmology, and it will be explicitly demonstrated that such a deformation removes the singularity in the FRW cosmology.

## I. INTRODUCTION

Even though it has not been possible to construct a quantum theory of gravity, there are many proposals for quantum gravity, and such proposals can have interesting physical consequences [1–4]. In fact, many of these proposals have predict similar physical consequences, and one such almost universal prediction of many different approach to quantum gravity is the deformation of the standard relativistic dispersion relation [5, 6]. Such a deformation of the standard relativistic dispersion relation occurs in various different approaches to quantum gravity, such as the spacetime discreteness [7], spacetime foam models [8], spontaneous symmetry breaking of Lorentz invariance in string field theory [9], and spin-network in Loop quantum gravity [10]. The standard energy-momentum dispersion relation gets deformed to a modified dispersion relation (MDR) near the Planck scale. It is possible to use MDR to explain certain astronomical and cosmological observations, such as the threshold anomalies of ultra high energy cosmic rays and TeV photons [11–17].

The MDR is based on the existence of a maximum energy scale, and it has been possible to construct a theory with such an intrinsic maximum energy scale [2, 3]. This theory is called the doubly special relativity, and in this theory the Planck energy ( $E_P$ ) and the velocity of light ( $c$ ) are two universally invariant quantities. Just as it is not possible for a particle to attain a velocity greater than the velocity of light in special relativity, it is not possible for a particle to attain an energy larger than the Planck energy in doubly special relativity. In the doubly special relativity, the Lorentz transformations are deformed to a set of nonlinear Lorentz transformation in momentum space. In fact, this deformation of the Lorentz transformations directly deforms the standard energy-momentum relation. It has been possible to extend the doubly special relativity to curved spacetime and obtain doubly general relativity [18]. In this theory, one assumes that the geometry of spacetime depends on the energy of the test particle. So, we do not have a single metric describing the geometry of spacetime, but a one-parameter family of energy dependent metrics. These metrics depend on the energy of the test particles. As we have a family of energy dependent metrics in such a theory, this theory is called the gravity's rainbow [8, 18].

Recently, gravity's rainbow has been used to study the high energy behavior of various physical systems [19–26]. The rainbow deformation of various black hole solutions have been performed, and their properties have been studied [27–33]. The hydrostatic equilibrium for compact objects and the structure of neutron stars has also been investigated using the gravity's rainbow [34, 35]. Furthermore, the effects of gravity's rainbow on wormholes have also been investigated [36]. The gravity's rainbow has also been used for analyzing the effects of rainbow functions on gravitational force and Starobinsky model of  $f(R)$  gravity [37, 38].

It may be noted that string theory can be regarded as two dimensional theory, and the target space metric can be regarded as a matrix of coupling constants for this two dimensional theory. These coupling constants will flow due to the renormalization group flow, and so the target space metric will depend on the scale at which spacetime is probed, but this scale would in turn depend on the energy of the test particle used to probe this spacetime. Thus, the target space metric in string theory would depend on the energy of the probe, and so gravity's rainbow is motivated from

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\* email address: hendi@shirazu.ac.ir

† email address: m.momennia@shirazu.ac.ir

‡ email address: behzad.eslampanah@gmail.com

§ email address: f2mir@uwaterloo.ca

string theory [18]. It may be noted that the low energy effective field theory approximation to the heterotic string theory [39, 40], produces the Gauss-Bonnet (GB) gravity [41, 42]. It has been demonstrated that the low-energy expansion of string theory effective action contains the GB term and a scalar field [43]. It is possible to neglect the effect of this scalar field as it can be regarded as a constant field. The GB gravity contains curvature-squared terms, and is free of ghosts. Furthermore, the corresponding field equations contain no more than second derivatives of the metric [44–47]. Black object solutions have also been studied in GB gravity [48–57]. As both GB gravity and gravity’s rainbow can be motivated from string theory, there is a strong motivation to study rainbow deformation of GB gravity. In fact, the thermodynamics of black holes has been studied using a combination of the gravity’s rainbow and GB gravity [58]. We also note that GB gravity has been used for analyzing various cosmological models [59–68]. The FRW cosmology in Einstein gravity’s rainbow has also been analyzed [69, 70]. It was observed that the universe is nonsingular in this model, however, it is important to analyze other cosmological models, so that we can know if this is a model dependent effect or a general feature of gravity’s rainbow. Furthermore, no work has been done on cosmological applications of GB-gravity’s rainbow, even though there are strong string theoretical motivations to perform this analysis. So, in this paper, we are going to analyze a cosmological model using GB-gravity’s rainbow. It will be observed that this model is also nonsingular, and so it seems that removal of big bang singularity because of the rainbow deformation is a general feature of rainbow deformation of any model of gravity.

## II. FRW RAINBOW COSMOLOGY IN EINSTEIN GRAVITY

Here, we are going to modify FRW universe in the Einstein gravity’s rainbow. We consider the Lagrangian of Einstein gravity with a matter field as

$$\mathcal{L}_E = \mathcal{R} + \mathcal{L}_m, \quad (1)$$

where  $\mathcal{R}$  is the Ricci scalar and  $\mathcal{L}_m$  is the Lagrangian of matter. Variation of the action (1) with respect to the metric tensor  $g_{\mu\nu}$  leads to

$$G_{\mu\nu}^E = 8\pi G T_{\mu\nu}, \quad (2)$$

where  $G_{\mu\nu}^E = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$  is the Einstein tensor. The energy-momentum tensor can be expressed as

$$T_{\mu\nu} = \rho u_\mu u_\nu + P(g_{\mu\nu} + u_\mu u_\nu), \quad (3)$$

where  $\rho$  and  $P$  are the energy density and the pressure of perfect fluid, respectively. Here  $u_\mu$  is defined as

$$u_\mu = (f^{-1}(\varepsilon), 0, 0, 0, 0), \quad (4)$$

and it is a unit vector

$$g^{\mu\nu} u_\mu u_\nu = -1. \quad (5)$$

Now, as we want to analyze this model using gravity’s rainbow, we will first review gravity’s rainbow. The gravity’s rainbow is based on the deformation of the standard energy-momentum dispersion relation,

$$E^2 f(\varepsilon)^2 - p^2 g(\varepsilon)^2 = m^2, \quad (6)$$

where  $\varepsilon = E/E_p$  and the functions  $f(\varepsilon)$  and  $g(\varepsilon)$  are called rainbow functions, and  $m$  is the mass of the test particle. In the IR limit, we have  $\lim_{\varepsilon \rightarrow 0} f(\varepsilon) = \lim_{\varepsilon \rightarrow 0} g(\varepsilon) = 1$  and so the standard energy-momentum dispersion relation is recovered in the IR limit of this theory. Thus, the gravity’s rainbow reduces to standard general relativity in the IR limit. As the Planck energy is the largest energy that a particle can attain, we can write

$$\varepsilon \leq 1. \quad (7)$$

The exact forms of the rainbow functions are constructed using various theoretical and observational motivations. In fact, the study that was done on the hard spectra from gamma-ray bursts has been used as a motivation to construct the following rainbow functions [8]

$$f(\varepsilon) = \frac{e^\varepsilon - 1}{\varepsilon} \quad \& \quad g(\varepsilon) = 1. \quad (8)$$

Now after substituting it in Eq. (6), we can find the corresponding MDR

$$p^2 = E^2 \left( \frac{e^\varepsilon - 1}{\varepsilon} \right)^2. \quad (9)$$

In order to compare the results of Einstein gravity with GB theory, one should reformulate them with identical dimensions. Since the GB term does not contribute in four dimensions, we consider the following five dimensional spacetime

$$ds^2 = -\frac{dt^2}{f(\varepsilon)^2} + \frac{R(t)^2}{g(\varepsilon)^2} dx_i^2, \quad i = 1, 2, 3, 4, \quad (10)$$

where  $R(t)$  is scale factor, and we consider a flat universe with  $k = 0$ . Using the above metric, field equation (2) and the energy-momentum tensor (3), the FRW equations in Einstein gravity's rainbow can be written as

$$\left( H - \frac{\dot{g}(\varepsilon)}{g(\varepsilon)} \right)^2 + \frac{\dot{g}(\varepsilon)}{g(\varepsilon)} \left( H - \frac{3\dot{g}(\varepsilon)}{4g(\varepsilon)} \right) = \frac{4\pi G\rho}{3f(\varepsilon)^2}, \quad (11)$$

$$g(\varepsilon) \left[ \dot{H} - \frac{\ddot{g}(\varepsilon)}{2g(\varepsilon)} - \frac{\dot{g}(\varepsilon)}{g(\varepsilon)} \left( H - \frac{\dot{g}(\varepsilon)}{g(\varepsilon)} \right) + \left( H + \frac{\dot{f}(\varepsilon)}{2f(\varepsilon)} \right) \left( 2H - \frac{\dot{g}(\varepsilon)}{g(\varepsilon)} \right) \right] \quad (12)$$

$$-2 \left( H - \frac{\dot{g}(\varepsilon)}{g(\varepsilon)} \right)^2 + \frac{\dot{g}(\varepsilon)}{g(\varepsilon)} \left( \frac{3\dot{g}(\varepsilon)}{2g(\varepsilon)} - 2H \right) = -\frac{8\pi G(\rho + P)}{3f(\varepsilon)^2}, \quad (13)$$

which  $H = \dot{R}(t)/R(t)$  is the Hubble parameter. It may be noted that we have used the notations  $\dot{A} = \frac{dA}{dt}$  and  $\ddot{A} = \frac{d^2A}{dt^2}$ . The conservation of energy-momentum tensor can be written as

$$\nabla_\mu T_\nu^\mu = \partial_\mu T_\nu^\mu - \Gamma_{\mu\nu}^\lambda T_\lambda^\mu + \Gamma_{\mu\lambda}^\mu T_\nu^\lambda = 0,$$

and this equation reduces to

$$\dot{\rho} + 2 \left( 2H - \frac{\dot{g}(\varepsilon)}{g(\varepsilon)} \right) (\rho + P) = 0. \quad (14)$$

We can consider a large range of ultra relativistic particles, which are in thermal equilibrium with an average energy  $\epsilon \sim T$ . The continuity equation leads to the first law of thermodynamics (as in standard cosmology)

$$d(\rho V) = -PdV, \quad (15)$$

where  $V$  is the volume and  $V = [R(t)/g(\varepsilon)]^4$ . The Eq. (15) along with the integrability condition  $\frac{\partial^2 S}{\partial V \partial P} = \frac{\partial^2 S}{\partial P \partial V}$  [71] leads to a constant entropy

$$S = \frac{V(\rho + P)}{T} = \text{const.} \quad (16)$$

In this paper, we are going to consider the following equation of state (EoS)

$$P = (\gamma - 1)\rho. \quad (17)$$

The FRW spacetime is singular at  $t = 0$ , if this EoS is used in the standard cosmology. In the above equation,  $\gamma$  is the EoS parameter. For this pressure, the average energy  $\epsilon$  can be written as

$$\epsilon \sim T = c\gamma\rho V, \quad (18)$$

where  $T$  is temperature and  $c$  is a constant (which is equal to  $1/S$ ). Using the EoS (17) in Eq. (14), we obtain the following equation

$$\frac{d\rho}{d\ln[R(t)^2/g(\varepsilon)]} = -2\gamma\rho, \quad (19)$$

which can be solved to give a density  $\rho = [R(t)^2/g(\varepsilon)]^{-2\gamma}$ . This leads to an average energy

$$\epsilon = \frac{c\gamma}{R(t)^4} \rho^{\frac{\gamma-2}{\gamma}}. \quad (20)$$

### III. MODIFIED FRW RAINBOW COSMOLOGY IN GB GRAVITY

Now, we are going to analyze the FRW universe using gravity's rainbow with GB term. We are also going to study its effect on the early universe using a semi-classical approximation. The Lagrangian of Einstein-GB gravity can be written as

$$\mathcal{L}_{tot} = \mathcal{R} + \alpha \mathcal{L}_{GB} + \mathcal{L}_m, \quad (21)$$

where the parameter  $\alpha$  in the second term of Eq. (21) is the GB coefficient with dimension  $(length)^2$ , and  $\mathcal{L}_{GB}$  is the Lagrangian of GB gravity

$$\mathcal{L}_{GB} = R_{\mu\nu\tau\sigma} R^{\mu\nu\tau\sigma} - 4R_{\mu\nu} R^{\mu\nu} + \mathcal{R}^2. \quad (22)$$

Variation of the action (21) with respect to the metric  $g_{\mu\nu}$  leads to

$$G_{\mu\nu}^E + \alpha G_{\mu\nu}^{GB} = 8\pi G T_{\mu\nu}, \quad (23)$$

where  $G_{\mu\nu}^{GB} = 2(R_{\mu\tau\sigma\lambda} R_{\nu}^{\tau\sigma\lambda} - 2R_{\mu\tau\nu\sigma} R^{\tau\sigma} - 2R_{\mu\lambda} R_{\nu}^{\lambda} + \mathcal{R} R_{\mu\nu}) - \frac{1}{2} \mathcal{L}_{GB} g_{\mu\nu}$ . Using the metric (10), field equation (23), and the energy-momentum tensor (3), we can write the FRW equations in gravity's rainbow with GB term as

$$\left(H - \frac{\dot{g}(\varepsilon)}{g(\varepsilon)}\right)^2 + \frac{\dot{g}(\varepsilon)}{g(\varepsilon)} \left(H - \frac{3\dot{g}(\varepsilon)}{4g(\varepsilon)}\right) + \alpha \frac{f(\varepsilon)^2}{8} \left(2H - \frac{\dot{g}(\varepsilon)}{g(\varepsilon)}\right)^4 = \frac{4\pi G \rho}{3f(\varepsilon)^2}, \quad (24)$$

$$\begin{aligned} g(\varepsilon) \left[ \dot{H} - \frac{\ddot{g}(\varepsilon)}{2g(\varepsilon)} - \frac{\dot{g}(\varepsilon)}{g(\varepsilon)} \left(H - \frac{\dot{g}(\varepsilon)}{g(\varepsilon)}\right) + \left(H + \frac{\dot{f}(\varepsilon)}{2f(\varepsilon)}\right) \left(2H - \frac{\dot{g}(\varepsilon)}{g(\varepsilon)}\right) \right] + \frac{\dot{g}(\varepsilon)}{g(\varepsilon)} \left(\frac{3\dot{g}(\varepsilon)}{2g(\varepsilon)} - 2H\right) \\ - 2 \left(H - \frac{\dot{g}(\varepsilon)}{g(\varepsilon)}\right)^2 + \frac{\alpha}{2} g(\varepsilon) f(\varepsilon)^2 \left(2H - \frac{\dot{g}(\varepsilon)}{g(\varepsilon)}\right)^2 \left[ 2(\dot{H} + H^2) + \frac{\dot{g}(\varepsilon)^2}{2g(\varepsilon)^2} + \left(1 - \frac{1}{2g(\varepsilon)}\right) \left(2H - \frac{\dot{g}(\varepsilon)}{g(\varepsilon)}\right)^2 \right. \\ \left. - \frac{\ddot{g}(\varepsilon)}{g(\varepsilon)} - \left(2H - \frac{\dot{f}(\varepsilon)}{f(\varepsilon)}\right) \left(2H - \frac{\dot{g}(\varepsilon)}{g(\varepsilon)}\right) \right] = -\frac{8\pi G(\rho + P)}{3f(\varepsilon)^2}. \end{aligned} \quad (25)$$

Here for  $\alpha = 0$ , Eqs. (24) and (25) reduce to Eqs. (11) and (13), respectively. The conservation equation for GB-gravity's rainbow can be written as

$$\dot{\rho} + 2 \left(2H - \frac{\dot{g}(\varepsilon)}{g(\varepsilon)}\right) (\rho + P) = 0. \quad (26)$$

It may be noted that Eq. (26) is the same as conservation equation obtained in Einstein gravity's rainbow, Eq. (14). Now, using the same procedure with Eqs. (17) and (26), it can be demonstrated that the average energy has the same form for GB gravity (20),

$$\epsilon = \frac{c\gamma}{R(t)^4} \rho^{\frac{\gamma-2}{\gamma}}. \quad (27)$$

### IV. WHEN A NONSINGULAR RAINBOW UNIVERSE IN GB GRAVITY IS POSSIBLE?

Before analyzing how the MDR (Eq. (9)) leads to a nonsingular cosmology, it is useful to discuss the general conditions on the rainbow functions that lead to a nonsingular universe. Substituting Eq. (17) and the modified Friedmann equation of gravity's rainbow (Eq. (24)) in the conservation equation (Eq. (26)), we obtain

$$\dot{\rho} = \pm 2\gamma \rho \left\{ \frac{8}{\alpha f(\varepsilon)^2} \left[ \frac{4\pi G \rho}{3f(\varepsilon)^2} - H \frac{\dot{g}(\varepsilon)}{g(\varepsilon)} - \left(H - \frac{\dot{g}(\varepsilon)}{g(\varepsilon)}\right)^2 \right] \right\}^{\frac{1}{4}}. \quad (28)$$

A similar system has been studied in Ref. [72], and this analysis was performed using the Hubble rate. However, it is also possible to study this model using density  $\rho$  instead of the Hubble rate  $H$  [69]. Our analysis will be based

on the approach used in Ref. [69], and we will demonstrate that this cosmological model is free from finite-time singularities. This is because an upper bound for the density  $\rho$  is reached in an infinite time. Thus, there is a point at which the density diverges, however, that point exists at an infinite time.

To use this explanation, we need a differential equation for  $\rho$  with respect to time. If we write  $f(\varepsilon)$  and  $g(\varepsilon)$  as functions of  $\rho$  and  $R$  instead of  $\varepsilon$  according to Eq. (27), the equation (28) will be too complicated, and we will not be able to solve it. So, to solve this problem, we choose the following form of the solution (separation of variables)

$$g(\varepsilon, t) = G(\varepsilon)R(t), \quad (29)$$

and it has the following consequence

$$\frac{\dot{g}(\varepsilon)}{g(\varepsilon)} = \frac{\dot{g}(\varepsilon, t)}{g(\varepsilon, t)} = \frac{G(\varepsilon)\dot{R}(t)}{G(\varepsilon)R(t)} = \frac{\dot{R}(t)}{R(t)} = H. \quad (30)$$

According to Eq. (29), the first deformed FRW equation (24) reduces to

$$H^2 + \frac{1}{2}f(\varepsilon)^2 H^4 \alpha = \frac{16\pi G\rho}{3f(\varepsilon)^2}. \quad (31)$$

In addition, considering Eq. (29), one can show that Eqs. (26) and (27) become

$$\dot{\rho} + 2H(\rho + P) = 0, \quad (32)$$

$$\epsilon = \frac{c\gamma}{g(\varepsilon)^4} \rho^{\frac{\gamma-2}{\gamma}}. \quad (33)$$

Now, substituting Eqs. (17) and (31) in Eq. (32), we obtain

$$\dot{\rho} = \pm 2\gamma\rho \left[ \frac{1}{\alpha f(\rho)^2} \left( \pm \sqrt{1 + \frac{32}{3}\pi G\rho\alpha} - 1 \right) \right]^{\frac{1}{2}}, \quad (34)$$

where we will choose the plus sign in parenthesis to obtain a consistent equation in Einstein gravity, i.e., in the limit  $\alpha \rightarrow 0$ . We expressed  $f$  as a function of  $\rho$  instead of  $\varepsilon$  according to Eq. (33). Now, one can show that finite-time singularities (including big bang singularity) are absent if  $f$  grows asymptotically as  $\rho^{1/4}$ , or faster. For example, if  $f \sim \rho^s$ , where  $s \geq 1/4$ . In this case, one can calculate the time for reaching a potential singularity by integrating Eq. (34) (starting from some initial finite density  $\rho^*$  to an infinite one). This integration leads to

$$t = \pm \frac{\sqrt{\alpha}}{2\gamma} \int_{\rho^*}^{\infty} \rho^{s-1} \left( \sqrt{1 + \frac{32}{3}\pi G\rho\alpha} - 1 \right)^{-\frac{1}{2}} d\rho. \quad (35)$$

After some calculations we obtain

$$t = \pm \frac{\rho^s \{2(16s^2 - 32s + 15)\mathcal{H}_1 + \varrho\}}{\gamma(4s-1)(4s-3)(4s-5)(1+\mathcal{X})^s} \sqrt{\frac{\alpha\mathcal{X}}{2}} \Big|_{\rho^*}^{\infty}, \quad (36)$$

where

$$\varrho = -(4s-1)\mathcal{X}[(4s-5)\mathcal{H}_2 - (4s-3)\mathcal{X}\mathcal{H}_3],$$

and  $\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3$  are the following hypergeometric functions

$$\mathcal{H}_1 = {}_2F_1 \left( \left[ -s, \frac{1}{2} - 2s \right], \left[ \frac{3}{2} - 2s \right], -\mathcal{X} \right),$$

$$\mathcal{H}_2 = {}_2F_1 \left( \left[ -s, \frac{3}{2} - 2s \right], \left[ \frac{5}{2} - 2s \right], -\mathcal{X} \right),$$

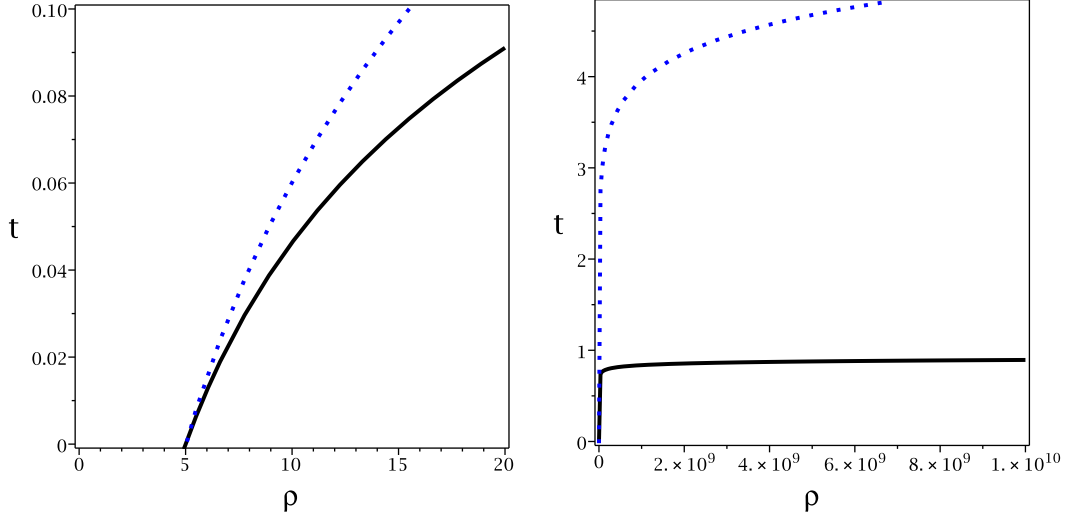


FIG. 1: **GB case:** Time versus density, for  $G = 2$ ,  $\gamma = 4/3$ ,  $\alpha = 0.1$ ,  $\rho^* = 5$ ,  $s = 1/5$  (continuous line) and  $s = 1/3$  (dotted line): "left and right figures indicate various  $\rho$  ranges".

$$\mathcal{H}_3 = {}_2F_1 \left( \left[ -s + 1, \frac{5}{2} - 2s \right], \left[ \frac{7}{2} - 2s \right], -\mathcal{X} \right),$$

with

$$\mathcal{X} = \frac{6}{\sqrt{9 + 96\pi G \rho \alpha} - 3}.$$

Comparing various terms in Eq. (36), one can show that time is infinite for  $s \geq 1/4$ . Thus, the time to reach the potential singularity is infinite, so it is not a finite-time singularity, i.e., not physical. For more clarification, we consider the term with  $\rho$  as dominant term in Eq. (35), so we have

$$t = \pm \frac{\sqrt{\alpha}}{2\gamma} \int_{\rho^*}^{\infty} \rho^{s-1} \left( \frac{32}{3} \pi G \rho \alpha \right)^{-\frac{1}{4}} d\rho = \pm \frac{2\sqrt{3\alpha}}{\gamma(4s-1)} (96\pi G \alpha)^{-\frac{1}{4}} \rho^{s-\frac{1}{4}} \Big|_{\rho^*}^{\infty} = \infty, \quad s \geq \frac{1}{4}, \quad (37)$$

and here we obtain the same result for  $s$ . We conclude that the rainbow function  $f(\varepsilon)$  plays an important role in possible resolution of the big bang singularity, but it has to grow asymptotically as  $\rho^{1/4}$ , or faster.

Now, we can discuss Eq. (37) and plot  $t - \rho$  diagram for  $s < 1/4$  and  $s > 1/4$  in Fig. 1. Considering this figure, one can find an initial finite density at  $t = 0$  (present time), as expected. In addition, for  $s < 1/4$ , we obtain a finite value for time (to backward), when the density of the universe goes to infinity (big bang singularity). However, in the case of  $s > 1/4$ , there is no finite (backward) time to obtain infinite density and therefore, there is no big bang singularity at any finite time in the past.

It is not necessary to perform similar analysis to investigate the rainbow deformation of FRW cosmology in Einstein gravity. To do so, it is sufficient to expand the function in Eq. (35) for  $\alpha \rightarrow 0$ . So, using rainbow deformation of the Einstein theory, we obtain

$$t = \pm \frac{1}{8\gamma} \sqrt{\frac{3}{\pi G}} \int_{\rho^*}^{\infty} \rho^{s-\frac{3}{2}} d\rho = \pm \frac{1}{4\gamma(2s-1)} \sqrt{\frac{3}{\pi G}} \rho^{s-\frac{1}{2}} \Big|_{\rho^*}^{\infty} = \infty, \quad s \geq \frac{1}{2}, \quad (38)$$

where it shows that the value of  $f$  for having a nonsingular universe in Einstein gravity has to grow asymptotically as  $\rho^{1/2}$ , or faster.

Here, we plot  $t - \rho$  diagram in Einstein case (Eq. (38)) for both  $s < 1/2$  and  $s > 1/2$  in Fig. 2. Like GB case, Fig. 2 shows that there is an initial finite density at  $t = 0$  (present time). There is no infinite density (big bang singularity) in a finite (backward) time for  $s > 1/2$ . However, for  $s < 1/2$ , there is a finite (backward) time at which an infinite density (big bang singularity) exists.

Here, we have investigated possibility of obtaining a nonsingular rainbow universe in the Einstein and GB gravities. In the coming section, we are going to use the MDR (Eq. (9)) to analyze such a nonsingular FRW-like cosmology.

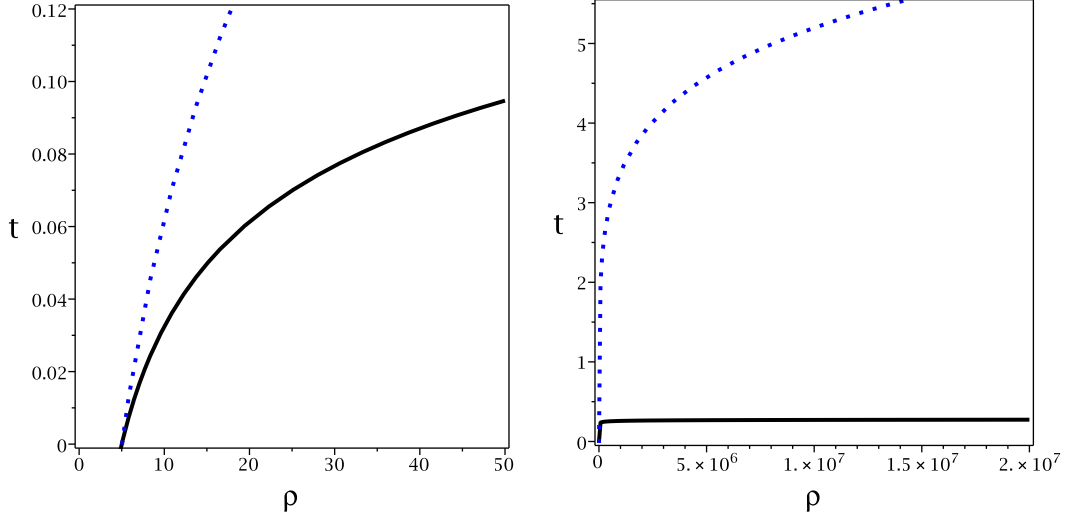


FIG. 2: **Einstein case:** Time versus density, for  $G = 2$ ,  $\gamma = 4/3$ ,  $\rho^* = 5$ ,  $s = 1/3$  (continuous line) and  $s = 2/3$  (dotted line): "left and right figures indicate various  $\rho$  ranges".

## V. NONSINGULAR RAINBOW UNIVERSES

Using Eqs. (8), (17), and (26), one can show that average energy (27) can be expressed as

$$\epsilon = c\gamma\rho^{\frac{\gamma-1}{\gamma}}. \quad (39)$$

Now using Eqs. (8) and (39), the function  $f(\epsilon)$  will be

$$f(\epsilon) = \frac{\exp\left(\gamma\mathcal{G}^{\frac{\gamma-1}{\gamma}}\right) - 1}{\gamma\mathcal{G}^{\frac{\gamma-1}{\gamma}}}, \quad (40)$$

where  $\mathcal{G} = \rho/\rho_P$ , and  $E_P = c\rho_P^{\frac{\gamma-1}{\gamma}}$  is the Planck energy versus density  $\rho_P$ . Using the above equation and the MDR relation, one can show that the modified Friedmann equation (24) will be given by

$$H = \pm \frac{\gamma\mathcal{G}^{\frac{\gamma-1}{\gamma}} \left[ -9 \pm 3\sqrt{9 + 96\pi\alpha\mathcal{G}\rho_P^{\frac{2-\gamma}{\gamma}}} \right]^{\frac{1}{2}}}{6\sqrt{\alpha} \left[ \exp\left(\gamma\mathcal{G}^{\frac{\gamma-1}{\gamma}}\right) - 1 \right]}. \quad (41)$$

We will choose plus sign in parenthesis to obtain a consistent equation in Einstein gravity, i.e., in the limit  $\alpha \rightarrow 0$ . We can investigate a possible singular solution of the big bang singularity using the discussion of section IV. Substituting Eq. (17) and the modified Friedmann equation (41) in Eq. (26) and using Eq. (8), we can obtain the following equation

$$\dot{\mathcal{G}} = \pm \frac{2\gamma^2\mathcal{G}^{\frac{2\gamma-1}{\gamma}} \left[ -9 + 3\sqrt{9 + 96\pi\alpha\mathcal{G}\rho_P^{\frac{2-\gamma}{\gamma}}} \right]^{\frac{1}{2}}}{3\sqrt{\alpha} \left[ \exp\left(\gamma\mathcal{G}^{\frac{\gamma-1}{\gamma}}\right) - 1 \right]}, \quad (42)$$

where  $\dot{\mathcal{G}} = \dot{\rho}/\rho_P$ .

Now, we want to show that the time is infinite when we go from an initial finite density  $\mathcal{G}^*$  to an infinite one in special case  $\gamma = 4/3$ , (i.e., radiation). This can be done by integrating Eq. (42)

$$t = \pm \frac{27\sqrt{\alpha}}{32} \int_{\mathcal{G}^*}^{\infty} \frac{\exp\left(\frac{4}{3}\mathcal{G}^{\frac{1}{3}}\right) - 1}{\mathcal{G}^{\frac{5}{4}} \left[ -9 + 3\sqrt{9 + 96\pi\alpha\mathcal{G}\rho_P^{\frac{1}{2}}} \right]^{\frac{1}{2}}} d\mathcal{G}, \quad (43)$$

in which it is too hard to compute this integration analytically; however one can use numerical calculation to show that it does not converge on  $[\mathcal{G}^*, \infty)$ , so the time to reach the infinite density is infinite. For more clarification, we consider the term with  $\mathcal{G}$  as dominant term in denominator, and so we obtain

$$\begin{aligned} t &= \pm \frac{27}{32} \left[ \frac{3}{\alpha} \sqrt{96\pi\alpha\rho_P^{\frac{1}{2}}} \right]^{-\frac{1}{2}} \int_{\mathcal{G}^*}^{\infty} \frac{\exp\left(\frac{4}{3}\mathcal{G}^{\frac{1}{4}}\right) - 1}{\mathcal{G}^{\frac{3}{2}}} d\mathcal{G} \\ &= \pm \frac{27}{32} \left[ \frac{3}{\alpha} \sqrt{96\pi\alpha\rho_P^{\frac{1}{2}}} \right]^{-\frac{1}{2}} \left( \frac{2}{\mathcal{G}^{\frac{1}{2}}} - \frac{32\mathcal{E}\left(1, -\frac{4}{3}\mathcal{G}^{\frac{1}{4}}\right)}{9} - \frac{2\mathcal{G}^{-\frac{1}{2}}\left(3 + 4\mathcal{G}^{\frac{1}{4}}\right)}{3\exp\left(\frac{-4\mathcal{G}^{\frac{1}{4}}}{3}\right)} \right) \Bigg|_{\mathcal{G}^*}^{\infty} = \infty, \end{aligned} \quad (44)$$

where  $\mathcal{E}$  is the exponential integration. This shows that the time to reach this infinite density is infinite. Thus, there is no finite-time singularities, and this result confirms the consequence of Eq. (43).

In order to investigate the rainbow deformation of the Einstein gravity, one can follow the same procedure using Eqs. (8), (11), (14) and (39) or just expand the function in integration (43) for  $\alpha \rightarrow 0$ . Here, we use the second way, and we obtain

$$t = \pm \frac{9}{128} \sqrt{\frac{3}{\pi\rho_P^{\frac{1}{2}}}} \int_{\mathcal{G}^*}^{\infty} \frac{\exp\left(\frac{4}{3}\mathcal{G}^{\frac{1}{4}}\right) - 1}{\mathcal{G}^{\frac{3}{4}}} d\mathcal{G} = \pm \frac{9}{128} \sqrt{\frac{3}{\pi\rho_P^{\frac{1}{2}}}} \left( \frac{4}{3\mathcal{G}^{\frac{3}{4}}} + \zeta \right) \Bigg|_{\mathcal{G}^*}^{\infty} = \infty, \quad (45)$$

where

$$\zeta = -\frac{128\mathcal{E}\left(1, -\frac{4}{3}\mathcal{G}^{\frac{1}{4}}\right)}{81} - \frac{4\left(9 + 8\mathcal{G}^{\frac{1}{2}} + 6\mathcal{G}^{\frac{1}{4}}\right)}{27\mathcal{G}^{\frac{3}{4}}\exp\left(\frac{-4}{3}\mathcal{G}^{\frac{1}{4}}\right)}.$$

This result shows that there is no finite-time singularities.

We also plot  $t - \rho$  diagram for both Einstein and GB gravities (Eqs. (44) and (45)) in Fig. 3. This figure shows that for both Einstein and GB gravities, there is an initial finite density at  $t = 0$ , however, (backward) time goes to infinity as density goes to infinity and therefore there is no big bang singularity.

Finally, it is interesting to investigate the behavior of density of states at the Planck scale to analyze its divergences [70, 73]. Using the MDR, the density of states can be written as

$$a(E)dE \simeq p^3 dp \simeq f(\varepsilon)^4 \left( 1 + E \frac{f(\varepsilon)'}{f(\varepsilon)} \right) E^3 dE. \quad (46)$$

Here by substituting the MDR and using the fact that energy cannot be larger than the Planck energy, the density of states has a finite value  $e(e-1)^3$  with regular behavior without any divergences.

## VI. CONCLUSIONS

In this work, we have investigated the effect of gravity's rainbow in Einstein and GB gravities for the early universe. We have analyzed the rainbow deformation of the 5-dimensional FRW solution in both Einstein and GB gravity. We have observed that although GB term contributes to the field equations, it does not change the conservation equation and average energy. We have also demonstrated that the rainbow functions modify both conservation equation and average energy. Then, we have discussed the general conditions for having the nonsingular FRW cosmologies using the rainbow deformation of both Einstein and GB gravities.

We have used the rainbow functions defined by Amelino-Camelia, et. al., [8, 74] to investigate the effect of rainbow deformation of FRW-like cosmology. We have demonstrated that it is possible to obtain nonsingular cosmological solutions by using a rainbow deformation of Einstein and GB gravities. The Friedmann equations were modified using the gravity's rainbow by suitable rainbow functions. We also identified the rainbow functions with the MDR introduced by Amelino-Camelia, et al. [8, 74], and studied the rainbow modified Friedmann equations of a perfect fluid. We have found nonsingular solutions for a wide range of values for the equation of state parameter  $\gamma > 4/3$  in both Einstein and GB gravities. We also found that GB gravity has considerable effect on the constraint for having nonsingular universes. Using the analysis done in [69], we found that the universe takes infinite time to reach  $\rho \rightarrow \infty$



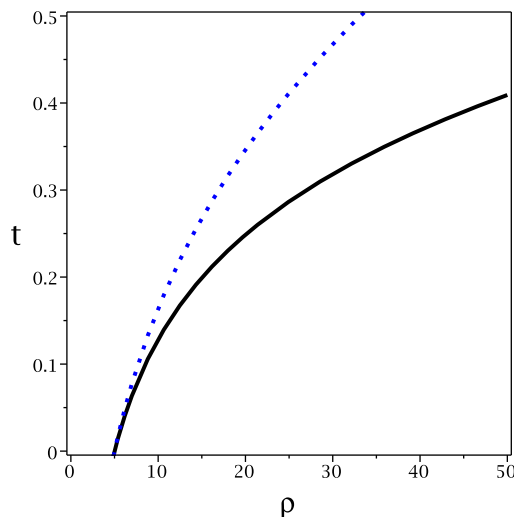


FIG. 3: Time versus density, for  $\mathcal{G}^* = 5$  and  $\rho_P = 0.2$ . Einstein gravity's rainbow ( $\alpha = 0$ : continuous line) and Gauss-Bonnet gravity's rainbow ( $\alpha = 0.1$ : dotted line).

from a finite value of  $\rho$ . We have also found that the density of states do not diverge at the Planck scale. So for both cases, we found a possible resolution of the big bang singularity. Hence, it seems that the removal of singularities by rainbow deformation is not a model dependent effect. It would be interesting to perform this analysis in other models of Lovelock gravity.

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- [1] G. Amelino-Camelia, Phys. Lett. B **510**, 255 (2001).
  - [2] G. Amelino-Camelia, Int. J. Mod. Phys. D **11**, 35 (2002).
  - [3] J. Magueijo and L. Smolin, Phys. Rev. D **67**, 044017 (2003).
  - [4] L. Smolin, Nucl. Phys. B **742**, 142 (2006).
  - [5] A. Kempf, G. Mangano and R. B. Mann, Phys. Rev. D **52**, 1108 (1995).
  - [6] F. Brau, J. Phys. A **32**, 7691 (1999).
  - [7] G. 't Hooft, Class. Quantum Gravit. **13**, 1023 (1996).
  - [8] G. Amelino-Camelia, J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos and S. Sarkar, Nature **393**, 763 (1998).
  - [9] V. A. Kostelecky and S. Samuel, Phys. Rev. D **39**, 683 (1989).
  - [10] R. Gambini and J. Pullin, Phys. Rev. D **59**, 124021 (1999).
  - [11] G. Amelino-Camelia, J. Lukierski and A. Nowicki, Phys. Atom. Nucl. **61**, 1811 (1998).
  - [12] D. Colladay and V. A. Kostelecky, Phys. Rev. D **58**, 116002 (1998).
  - [13] M. Takeda et al., ApJ. **522**, 225 (1999).
  - [14] D. Finkbeiner, M. Daris and D. Schlegel, ApJ. **544**, 81 (2000).
  - [15] R. C. Myers and M. Pospelov, Phys. Rev. Lett. **90**, 211601 (2003).
  - [16] T. A. Jacobson, S. Liberati, D. Mattingly and F. W. Stecker, Phys. Rev. Lett. **93**, 021101 (2004).
  - [17] G. Amelino-Camelia, Living. Rev. Relativ. **16**, 5 (2013).
  - [18] J. Magueijo and L. Smolin, Class. Quantum Gravit. **21**, 1725 (2004).
  - [19] P. Galan and G. A. Mena Marugan, Phys. Rev. D **70**, 124003 (2004).
  - [20] J. Hackett, Class. Quantum Gravit. **23**, 3833 (2006).
  - [21] R. Aloisio, A. Galante, A. Grillo, S. Liberati, E. Luzio and F. Mendez, Phys. Rev. D **73**, 045020 (2006).
  - [22] Y. Ling, X. Li and H. Zhang, Mod. Phys. Lett. A **22**, 2749 (2007).

- [23] R. Garattini and B. Majumder, Nucl. Phys. B **884**, 125 (2014).
- [24] Z. Chang and S. Wang, Eur. Phys. J. C **75**, 259 (2015).
- [25] G. Santos, G. Gubitosi and G. Amelino-Camelia, JCAP **08**, 005 (2015).
- [26] A. F. Ali and M. M. Khalil, Europhys. Lett. **110**, 20009 (2015).
- [27] P. Galan and G. A. Mena Marugan, Phys. Rev. D **74**, 044035 (2006).
- [28] A. F. Ali, Phys. Rev. D **89**, 104040 (2014).
- [29] Y. Gim and W. Kim, JCAP **05**, 002 (2015).
- [30] Y. Gim and W. Kim, Eur. Phys. J. C **76**, 166 (2016).
- [31] B. Mu, P. Wang and H. Yang, JCAP **11**, 045 (2015).
- [32] S. H. Hendi, M. Faizal, B. Eslam Panah and S. Panahiyan, Eur. Phys. J. C **76**, 296 (2016).
- [33] S. H. Hendi, S. Panahiyan, B. Eslam Panah and M. Momennia, Eur. Phys. J. C **76**, 150 (2016).
- [34] S. H. Hendi, G. H. Bordbar, B. Eslam Panah and S. Panahiyan, JCAP **09**, 013 (2016).
- [35] R. Garattini and G. Mandanici, [arXiv:1601.00879].
- [36] R. Garattini and F. S. N. Lobo, [arXiv:1512.04470].
- [37] A. S. Sefiedgar, Int. J. Mod. Phys. D **25**, 1650101 (2016).
- [38] A. Chatrabhuti, V. Yingcharoenrat and P. Channuie, Phys. Rev. D **93**, 043515 (2016).
- [39] D. J. Gross and E. Witten, Nucl. Phys. B **277**, 1 (1986).
- [40] R. R. Metsaev and A. A. Tseytlin, Phys. Lett. B **191**, 354 (1987).
- [41] K. S. Stelle, Gen. Relativ. Gravit. **9**, 353 (1978).
- [42] J. W. Maluf, Gen. Relativ. Gravit. **19**, 57 (1987).
- [43] R. R. Metsaev and A. A. Tseytlin, Phys. Lett. B **185**, 52 (1987).
- [44] D. G. Boulware and S. Deser, Phys. Rev. Lett. **55**, 2656 (1985).
- [45] B. Zumino, Phys. Rept. **137**, 109 (1986).
- [46] Jr. C. G. Callan, R. C. Myers and M. J. Perry, Nucl. Phys. B **311**, 673 (1989).
- [47] R. G. Cai, Phys. Rev. D **65**, 084014 (2002).
- [48] R. C. Myers and J. Z. Simon, Phys. Rev. D **38**, 2434 (1988).
- [49] Y. M. Cho and I. P. Neupane, Phys. Rev. D **66**, 024044 (2002).
- [50] S. Mignemi, Phys. Rev. D **74**, 124008 (2006).
- [51] C. M. Chen, D. V. Gal'tsov and D. G. Orlov, Phys. Rev. D **78**, 104013 (2008).
- [52] C. Bogdanos, C. Charmousis, B. Gouteraux and R. Zegers, JHEP **10**, 037 (2009).
- [53] S. H. Hendi and B. Eslam Panah, Phys. Lett. B **684**, 77 (2010).
- [54] R. G. Cai, Y. Liu and Y. W. Sun, JHEP **04**, 090 (2010).
- [55] Y. Brihaye, B. Kleihaus, J. Kunz and E. Radu, JHEP **11**, 098 (2010).
- [56] M. B. Gaete and M. Hassaine, Phys. Rev. D **88**, 104011 (2013).
- [57] D. Ayzenberg and N. Yunes, Phys. Rev. D **90**, 044066 (2014).
- [58] S. H. Hendi and M. Faizal, Phys. Rev. D **92**, 044027 (2015).
- [59] N. Deruelle and T. Dolezel, Phys. Rev. D **62**, 103502 (2000).
- [60] K. H. Kim and Y. S. Myung, JCAP **12**, 004 (2004).
- [61] L. Amendola, C. Charmousis and S. C. Davis, JCAP **12**, 020 (2006).
- [62] B. M. Leith and I. P. Neupane, JCAP **05**, 019 (2007).
- [63] E. Elizalde, A. N. Makarenko, V. V. Obukhov, K. E. Osetrin and A. E. Filippov, Phys. Lett. B **644**, 1 (2007).
- [64] Y. Brihaye and E. Radu, JHEP **09**, 006 (2008).
- [65] R. Chingangbam, M. Sami, P. V. Tretyakov and A. V. Toporensky, Phys. Lett. B **661**, 162 (2008).
- [66] K. Bamba, A. N. Makarenko, A. N. Myagky and S. D. Odintsov, Phys. Lett. B **732**, 349 (2014).
- [67] S. Capozziello, M. Francaviglia and A. N. Makarenko, Astrophys. Space Sci. **349**, 603 (2014).
- [68] P. Kanti, R. Gannouji and N. Dadhich, Phys. Rev. D **92**, 083524 (2015).
- [69] A. Awad, A. F. Ali and B. Majumder, JCAP **10**, 052 (2013).
- [70] Y. Ling, JCAP **08**, 017 (2007).
- [71] E. W. Kolb and M. S. Turner, Front. Phys. **69**, 1 (1990).
- [72] A. Awad, Phys. Rev. D **87**, 103001 (2013).
- [73] Y. Ling and Q. Wu, Phys. Lett. B **687**, 103 (2010).
- [74] G. Amelino-Camelia, New J. Phys. **6**, 188 (2004).